Respuestas a la evaluación de la competencia del capítulo 2

Identifica el método de integración más apropiado para resolver la integral indefinida dada y resuélvela.

1. $\int 5x(1+x^2)^3\,dx = \frac{5}{8}\left(1+x^2\right)^4 + C$

$u = 1+x^2 \qquad du = 2xdx$

2. $\int e^{2x}\sqrt[3]{10-e^{2x}}\,dx = -\frac{3}{8}\left(10-e^{2x}\right)^{4/3} + C$

$u = 10-e^{2x} \qquad du = -2e^{2x}dx$

3. $\int \frac{x^3}{1+x^4}dx = \frac{1}{4}\ln\left(1+x^4\right)+C$

$u = 1+x^4 \qquad du = 4x^3dx$

4. $\int \frac{\operatorname{sen}\theta d\theta}{1-\cos\theta} = \ln\left|1-\cos(\theta)\right| + C$

$u = 1-\cos(\theta) \qquad du = \operatorname{sen}(\theta)d\theta$

5. $\int \frac{x^2+1}{x+1}dx = \frac{x^2}{2} - x + 2\ln\left|x+1\right| + C$

$\frac{x^2+1}{x+1} = x-1+\frac{2}{x+1}$

6. $\int \operatorname{sen} t \ln\left|\cos t\right| dt = -\cos t \ln\left|\cos t + \cos t + C\right|$

$u = \cos t \qquad du = -\operatorname{sen} t dt$

7. $\int \tan(x/3)\,dx = -3\ln\left|\cos\left(\frac{x}{3}\right)\right| + C$

$u = \frac{x}{3} \qquad du = \frac{dx}{3}$

8. $\int x\sec^2(x^2)\,dx = \frac{1}{2}\tan\left(x^2\right) + C$

$u = x^2 \qquad du = 2xdx$

9. $\int \tan^2(2\theta - a)\,d\theta = \frac{1}{2}\tan\left(2\theta - a\right) - \frac{2\theta - a}{2} + C$

$u = 2\theta - a \qquad du = 2d\theta$

10. $\int e^x \operatorname{sen}(e^x)\,dx = -\cos\left(e^x\right) + C$

$u = e^x \qquad du = e^x dx$

11. $\int \frac{dy}{\sqrt{1+25y^2}} = \frac{1}{5}\ln\left(\sqrt{1+25y^2} + 5y\right) + C$

$5y = \tan\left(\theta\right)$

$dy = \frac{1}{5}\sec^2\left(\theta\right)d\theta$

$\sqrt{1+25y^2} = \sec\left(\theta\right)$

12. $\int \frac{dx}{x\sqrt{1+x^2}} = \ln\left|\left[\frac{\sqrt{1+x^2} - 1}{x}\right] + C\right|$

$\sqrt{1+x^2} = \sec\left(\theta\right)$

$x = \tan\left(\theta\right) \qquad dx = \sec^2\left(\theta\right)d\theta$

13. $\displaystyle\int \frac{\tanh(\sqrt{x})dx}{\sqrt{x}} = 2\ln\left|\cosh\sqrt{x}\right| + C$

$$u=\sqrt{x} \qquad du=\frac{dx}{2\sqrt{x}}$$

14. $\displaystyle\int \operatorname{senh}(e^x)e^x\,dx = \cosh\left(e^x\right)+C$

$$u=e^x \qquad du=e^x dx$$

15. $\displaystyle\int \frac{dx}{\operatorname{sen}(x)+\cos(x)} = \sqrt{2}\ln\left[\frac{\sqrt{2}\operatorname{sen}(x)-1}{\sqrt{2}\operatorname{sen}(x)+1}\right]-\sqrt{2}\ln\left[\frac{\sqrt{2}\cos(x)-1}{\sqrt{2}\cos(x)+1}\right]+C$

16. $\displaystyle\int \frac{x}{e^x}dx = \int \frac{x}{e^x}dx = \int xe^{-x}\,dx = -xe^{-x}+\int e^{-x}\,dx = -\frac{x}{e^x}-\frac{1}{e^x}+C = -\frac{1}{e^x}\left(x+1\right)+C$

$$u=x \qquad du=dx$$
$$v=-e^{-x} \qquad dv=e^x dx$$

17. $\displaystyle\int x^2\cosh(2x)\,dx = \frac{x^2}{2}\operatorname{senh}(2x)-\frac{x}{2}\cosh(2x)+\frac{1}{4}\operatorname{senh}(2x)+C$

$$u=x^2 \qquad du=2xdx \qquad\qquad u=x \qquad du=dx$$
$$dv=\cosh(2x)dx \qquad v=\frac{1}{2}\operatorname{senh}dv \qquad dv=\operatorname{senh}(2x)dx \qquad v=\frac{1}{2}\cosh(2x)$$

18. $\displaystyle\int x\ln\left(x^2\right)dx = x^2\ln x-\frac{1}{2}x^2+C$

$$u=\ln x \qquad du=\frac{1}{x}dx$$
$$dv=2xdx \qquad v=x^2$$

19. $\int \arctan(y)\,dy = y\arctan(y) - \frac{1}{2}\ln\left|1+y^2\right| + C$

$$u = \arctan(y) \qquad du = \frac{dy}{1+y^2}$$
$$dv = dy \qquad v = y$$

20. $\int x^2\left[\ln(x)\right]^2 dx = \frac{x^3}{3}\left\{\left[\ln(x)\right]^2 - \frac{2}{3}\left[\ln(x)\right] + \frac{2}{9}\right\} + C$

$$x = e^z \qquad u = z^2 \qquad du = 2z\,dz \qquad u = z \qquad du = dz$$
$$dx = e^z\,dz \qquad dv = e^{3z}dz \qquad v = \frac{1}{3}e^{3z} \qquad dv = e^{3z}dz \qquad v = \frac{1}{3}e^{3z}$$

21. $\int \text{sen}^3(x)\,dx = \frac{\cos^3(x)}{3} - \cos(x) + C$

22. $\int \cos^3(4z)\,\text{sen}(4z)\,dz = -\frac{1}{16}\cos^4(4z) + C$

$$u = \cos(4z) \qquad du = -4\,\text{sen}(4z)\,dz$$

23. $\int \tan^5(y)\,dy = \frac{1}{4}\sec^4(y) - \frac{1}{2}\sec^2(y) - \ln\left|\cos(y)\right| + C$

24. $\int \cot(\theta)\csc^6(\theta)\,d\theta = -\frac{1}{6}\csc^6(\theta) + C$

$$u = \text{sen}(\theta) \qquad du = \cos(\theta)\,d\theta$$

25. $\int \cos(x)\,\text{sen}(6x)\,dx = -\frac{1}{10}\cos(5x) - \frac{1}{14}\cos(7x) + C$

26. $\int \frac{x^3}{\sqrt{1-x^2}}dx = -\frac{(x+2)}{3}\bullet(1-x^2)^{1/2}+C$

$$\sqrt{1-x^2}=\cos(\theta)d\theta$$
$$x=\text{sen}(\theta)$$
$$dx=\cos(\theta)d\theta$$

27. $\int \frac{t^2dt}{\sqrt{t^2+2}} = \frac{1}{2}t\sqrt{t^2+2}-\ln\left[t+\sqrt{t^2+2}\right]+C$

$$u=t \qquad du=dt$$
$$dv=\frac{tdt}{\sqrt{t^2+2}} \qquad v=(t^2+2)^{1/2}$$

28. $\int \frac{\sqrt{t^2-8}}{t}dt = \sqrt{t^2-8}-\sqrt{8}\arctan\left(\sqrt{\frac{t^2-8}{8}}\right)+C$

29. $\int \frac{2x^2-5}{x^3-x}dx = \ln\left|\frac{x^5}{x^2-1}\right|+c$

30. $\int \frac{x}{(x-1)(x+1)^2}dx = \frac{1}{4}\ln\left[\frac{x-1}{x+1}\right]-\frac{1}{2}\bullet\ \frac{1}{(x+1)}+C$

31. $\int \frac{(-2t^2+3t-1)dt}{(t-1)(t^2+2)} = -\ln\left[t^2+2\right]+\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)+C$

32. $\int \frac{dx}{(x^2+1)^2} = \frac{1}{2}\left[\arctan(x)+\frac{x}{x^2+1}\right]+C$

$$x^2+1=\sec^2(\theta)$$
$$x=\tan(\theta)$$
$$dx=\sec^2(\theta)d\theta$$

33. $\int \sqrt{1+e^x}\,dx = 2\sqrt{1+e^x} + \ln\left[\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right] + C$

$$1+e^x = z^2$$
$$e^x dx = 2zdz$$
$$dx = \frac{2zdz}{z^2-1}$$

34. $\int \frac{ydy}{y^2+5y+6} = \ln\left(\frac{(y+3)^3}{(y+2)^2}\right) + C$

$$\frac{y}{y^2+5y+6} = \frac{y}{(y+3)(y+2)} = \frac{A}{(y+3)} + \frac{B}{(y+2)} = \frac{3}{(y+3)} + \frac{-2}{(y+2)}$$

35. $\int \frac{(9-x)}{(x+3)^3}dx = -\ln(x+3) - \frac{12}{x+3} + C$

$$\frac{9-x}{(x+3)^2} = \frac{A}{(x+3)} + \frac{B}{(x+3)^2} = \frac{-1}{(x+3)} + \frac{12}{(x+3)^2}$$

36. $\int \frac{(1+x)dx}{\sqrt{1-x^2}} = \operatorname{sen}^{-1}(x) - \sqrt{1-x^2} + C$

37. $\int \frac{x^3+x^2+x+2}{x^4+3x^2+2}dx = \arctan(x) + \frac{1}{2}\ln(x^2+2) + C$

$$\frac{x^3+x^2+x+2}{(x^2+1)(x^2+2)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+2} = \frac{1}{x^2+1} + \frac{x}{x^2+2}$$

38. $$\int \frac{dy}{\sqrt{y^2-2y-1}} = \ln\left(y-1+\sqrt{y^2-2y-1}\right)+C$$

$$u^2=(y-1)^2 \qquad u=y-1 \qquad du=dy$$
$$a^2=2; a=\sqrt{2}$$

39. $$\int \frac{dx}{e^x\sqrt{3+e^{2x}}} = -\frac{1}{3}\bullet\frac{\sqrt{3+e^{2x}}}{e^x}+C$$

$$x=\ln(z) \qquad dx=\frac{dz}{z}$$
$$z=e^x$$

$$\sqrt{3+z^2}=\sqrt{3}\sec(\theta)$$
$$z=\sqrt{3}\tan(\theta)$$
$$dz=\sqrt{3}\sec^2(\theta)d\theta$$

$$u= \qquad \theta$$
$$du=\cos(\quad)d$$

40. $$\int \frac{\cos(t)dt}{\operatorname{sen}^2(t)-2\operatorname{sen}(t)-8} = \frac{1}{6}\ln\left[\frac{\operatorname{sen}(t)-4}{\operatorname{sen}(t)+2}\right]+C$$

$$u^2=[\operatorname{sen}(t)-1]^2 \qquad u=\operatorname{sen}(t)-1 \qquad du=\cos(t)dt$$
$$a^2=9 \qquad a=3$$

41. $$\int \frac{x^4}{(1-x)^3}dx = -\frac{x^2}{2}-3x-6\ln(x-1)+\frac{8x-7}{2(x-1)^2}+C$$

$$\frac{x^4}{(x-1)^3}=x+3+\frac{6x^2-8x+3}{(x-1)^3}$$
$$=x+3+\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}$$
$$=x+3+\frac{6}{x-1}+\frac{4}{(x-1)^2}+\frac{1}{(x-1)^3}$$

42. $$\int \frac{x}{\sqrt{5x-x^2}}dx = -\left(5x-x^2\right)^{1/2} + \frac{5}{2}\ln\left(x-\frac{5}{2}+\sqrt{5x-x^2}\right)+C$$

$$u=-x^2+5x$$
$$du=\left(-2x+5\right)dx$$

$$u_1=5x-x^2$$
$$du_1=\left(5-2x\right)dx$$

$$u_2=\left(x-5/2\right)^2;a^2=25/4$$
$$u_2=x-5/2;a=5/2$$
$$du_2=dx$$

43. $$\int \tan\left(x\right)\sec^{3/2}\left(x\right)dx=\frac{2}{3}\sec^{3/2}\left(x\right)+C$$

$$u=\sec\left(x\right)$$
$$du=\sec\left(x\right)\tan\left(x\right)dx$$

44. $$\int \sec^9\left(\pi\theta\right)d\theta=\frac{\sec^7\left(\pi\theta\right)\tan\left(\pi\theta\right)}{8\pi}+\frac{7\sec^5\left(\pi\theta\right)\tan\left(\pi\theta\right)}{48\pi}+\frac{35\sec^3\left(\pi\theta\right)\tan\left(\pi\theta\right)}{192\pi}+$$
$$\frac{105\sec\left(\pi\theta\right)\tan\left(\pi\theta\right)}{384\pi}+\frac{105\ln\left[\sec\left(\pi\theta\right)+\tan\left(\pi\theta\right)\right]}{384}$$

45. $$\int \frac{xdx}{\left(x+10\right)^{14}} = -\frac{13x+10}{156\left(x+10\right)^{13}}+C$$

$$u=x \qquad du=dx$$
$$dv=\frac{dx}{\left(x+10\right)^{14}} \qquad v=-\frac{1}{13\left(x+10\right)^{13}}$$

46. $$\int \frac{dx}{(1+x)(1+x^2)^2} = \frac{1}{4}\ln\left(x+1\right)-\frac{1}{8}\ln\left(x^2+1\right)+\frac{1}{2}\arctan\left(x\right)+\frac{1}{4}\bullet \quad \frac{1}{x^2+1}+\frac{x}{4\left(x^2+1\right)}+C$$

47. $$\int \tan^6\left(y\right)\sec^4\left(y\right)dy =$$

$$u=\tan\left(y\right)$$
$$du=\sec^2\left(y\right)dy$$

$$=\frac{\tan^9\left(y\right)}{9}+\frac{\tan^7\left(y\right)}{7}+C$$

48. $\int \operatorname{sen}(t)\arctan\left[\operatorname{sen}^2(t)\right]\cos(t)\,dt = \frac{1}{2}\left\{\operatorname{sen}^2(t)\arctan\left[\operatorname{sen}^2(t)\right] - \frac{1}{2}\ln\left[1+\operatorname{sen}^4(t)\right]\right\} + C$

$z = \operatorname{sen}^2(t) \qquad dz = 2\operatorname{sen}(t)\cos(t)\,dt$

$u = \arctan(z) \qquad du = \frac{dz}{1+z^2}$
$dv = dz \qquad v = z$

49. $\int \ln\left(1+x^2\right)dx = x\ln\left(x^2+1\right) - 2x + 2\arctan(x) + C$

$u = \ln\left(x^2+1\right) \qquad du = \frac{2x\,dx}{x^2+1}$
$dv = dx \qquad v = x$

50. $\int \csc^5(\theta)\cot(\theta)\,d\theta = \frac{\left[\operatorname{sen}(\theta)\right]^{-5}}{-5} + C = -\frac{1}{5}\csc^5(\theta) + C$

$u = \operatorname{sen}(\theta) \qquad du = \cos(\theta)\,d\theta$

51. $\int x^3(6-x)^{10}\,dx = (x-6)^{11}\left[\frac{x^3}{11} - \frac{3x^2(x-6)}{11\bullet 12} + \frac{6x(x-6)^2}{11\bullet 12\bullet 13} - \frac{6(x-6)^3}{11\bullet 12\bullet 13\bullet 14}\right] + C$

52. $\int \operatorname{sen}^4(\pi n x)\,dx = -\frac{\operatorname{sen}^3(\pi n x)\cos(\pi n x)}{4} + \frac{3}{8}x - \frac{3}{16\pi n}\operatorname{sen}(2\pi n x) + C$

53. $\int \frac{\left(x^3+3\right)dx}{\left(x^2-1\right)\left(2x^2+1\right)} = \frac{2}{3}\ln(x-1) - \frac{1}{6}\ln(x+1) - \frac{5}{12}\ln\left(2x^2+1\right) - \frac{5}{24}\sqrt{2}\arctan\left(\sqrt{2}x\right) + C$

$\frac{\left(x^3+3\right)}{(x-1)(x+1)\left(2x^2+1\right)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{2x^2+1} = \frac{2/3}{x-1} - \frac{1/6}{x+1} - \frac{5/3\,x + 13/6}{2x^2+1}$

54. $\int \frac{\ln\left[\arctan(x)\right]dx}{1+x^2} = \arctan(x)\ln\left[\arctan(x)\right] - \arctan(x) + C$

$u = \arctan(x) \qquad du = \frac{dx}{1+x^2}$

55. $\int \cot^{1/2}(t)\csc^4(t)\,dt = -\frac{2}{3}\cot^{3/2}(t) - \frac{2}{7}\cot^{7/2}(t) + C$

$$u = \cot(t)$$
$$du = -\csc^2(t)\,dt$$

56. $\int \frac{\ln(x)+1}{x\ln(x)}\,dx = \ln(x) + \ln|\ln(x)+1| + C$

$$u = \ln(x)+1 \qquad du = \frac{dx}{x}$$

57. $\int \frac{\sec(x)}{\tan(x)+2\cot(x)} = -\arctan[\cos(x)] + C$

$$u^2 = \cos^2(x)$$
$$u = \cos(x)$$
$$du = -\operatorname{sen}(x)\,dx$$

58. $\int \frac{2x^4-3x^3-5x^2-4}{x^2(x^2-4)}\,dx = 2x - \frac{1}{x} - \ln(x-2) - 2\ln(x+2) + C$

59. $\int \frac{dy}{y^2\sqrt{64-y^2}} = -\frac{1}{64} \bullet \frac{\sqrt{64-y^2}}{y} + C$

60. $\int \frac{x^2}{7-x^6}\,dx = \frac{1}{3}\ \frac{1}{2\sqrt{7}}\ln\left(\frac{x^3+\sqrt{7}}{x^3-\sqrt{7}}\right) + C$

$$u^2 = x^6$$
$$u = x^3 \qquad du = 3x^2\,dx$$
$$a^2 = 7 \qquad a = \sqrt{7}$$

61. $\int \frac{1+x-x^2}{\sqrt{(1-x^2)^3}}\,dx = \operatorname{arcsen}(x) + \frac{1}{\sqrt{1-x^2}} + C$

$$u = 1-x^2$$
$$du = -2x\,dx$$

62. $$\int \sqrt{\frac{1-x}{x+1}}\,dx = \sqrt{1-x^2} + \operatorname{arcsen}\left(\sqrt{\frac{x+1}{2}}\right) + C$$

$$\boxed{\begin{aligned} &z^2 = x+1 \\ &x = z^2 - 1 \\ &dx = 2z\,dz \end{aligned}}$$

$$\boxed{\begin{aligned} &u^2 = z^2; a^2 = 2 \\ &u = z; a = \sqrt{2} \\ &du = dz \end{aligned}}$$

63. $$\int \frac{x + \arccos^2(3x)}{\sqrt{1-9x^2}}\,dx = \boxed{\begin{aligned} &u_1 = 1-9x^2 \qquad du_1 = -18x\,dx \\ &u_2 = \arccos(3x) \qquad du_2 = \frac{3dx}{\sqrt{1-9x^2}} \end{aligned}}$$

$$= -\frac{1}{9}\left(1-9x^2\right)^{1/2} + \frac{1}{9}\arccos(3x) + C$$

64. $$\int \cos(x)\operatorname{sen}(3x)\,dx = -\frac{1}{4}\cos(2x) - \frac{1}{8}\cos(4x) + C$$

66. $$\int \frac{1-\operatorname{sen}(x)}{\cos(x)}\,dx = \ln\left[1+\operatorname{sen}(x)\right] + C$$

67. $$\int \left[\tan^2(x) + \tan^4(x)\right] dx = \frac{\tan^3(x)}{3} + C$$

$$\boxed{\begin{aligned} &u = \tan(x) \\ &du = \sec^2(x)\,dx \end{aligned}}$$

68. Identidades trigonométricas

$$\int \sqrt{1+\cos(t)}\,dt = \begin{cases} 2 \bullet \sqrt{2}\operatorname{sen}\left(\dfrac{t}{2}\right) + C; -\pi < t < \pi \\ -2 \bullet \sqrt{2}\operatorname{sen}\left(\dfrac{t}{2}\right) + C; \pi < t < 2\pi \end{cases}$$